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FRACTAL NATURE OF THE SEA ICE DRAFT PROFILE

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Abstract. The fractal dimension is examined as a descriptor of ice roughness for more than 3000 km of under-ice draft submarine sonar data. The data can be considered to constitute a fractal set within a limited range of scales, as defined by the Hurst parameter H. It was found that $0.55 < H < 0.78$ for scales of 3-15 m and $0.15 < H < 0.45$ for scales of 15-75 m, beyond which H is near unity. From this it is seen quantitatively that sea ice on the large scale is smooth. The fractal dimension, $D = 2 - H$, at the smaller scales is similar to that measured by other investigators for individual ice features such as keels. The fractal dimension did not show any changing spatial pattern across ice regions, indicating that the scaling relationship is similar even when first-order measures such as the mean and variance of ice draft change. Therefore, D does not appear to be useful for partitioning the transect into homogeneous ice areas in the draft data examined.

1. Introduction

The under-ice shape and roughness influence acoustic scattering, heat fluxes, and friction, and are therefore important in geophysical studies of sea ice. Under-ice roughness can be described by first order statistics of the draft profile such as the range, variance and the spacing between keels, or with more complex measures such as the shape of the power spectrum and the fractal dimension. Fractal shapes are self-similar and independent of scale and therefore have no characteristic sizes as do Euclidean shapes (e.g., the radius of a sphere, the side of a cube); mathematical fractals have detail on all scales. Commonly used examples of fractal shapes include cloud edges and coastlines, where viewing at increasing magnification reveals more detail such that their lengths are undefinable. Additionally, they are often self-similar, where any portion viewed can be considered as a reduced-scale image of the whole.

The under-ice shape exhibits characteristics of self-similarity in the sense that it is comprised of level areas and ridges (called "keels" on the underside of the ice) which are in turn comprised of smaller level areas and ridges. For this reason we hypothesize that under-ice shape is fractal. Because fractal geometry, in which probability distributions are invariant with respect to scale, only applies over limited ranges of scale, we explore the fractal nature of the ice draft profile at different scales. Here a data set is defined as being "fractal" only when its fractal dimension, D, is strictly larger than the topological dimension; for a profile of ice draft - a line with a topological dimension of 1 - when $1 < D \leq 2$. We then examine the variability in this measure from region to region where other ice characteristics, such as keel depths and spacings, change.

Rothrock and Thorndike [1980] (hereafter RT80) examined a number of measures of roughness, among them was the slope of the power spectrum or spectral exponent, p, which is related to the fractal dimension by $D = 2 + (p + 1)/2$, $(-3 \leq p \leq -1)$. They examined data acquired by the nuclear submarine *USS Gurnard* during its April 1976 cruise in the Beaufort Sea. The sampling interval was roughly 1.5 m, and data were averaged over four points thereby increasing the interval to 6 m. The asymptotic spectral exponent (i.e., the slope for high frequencies/small wavelengths, approximately 12-100 m in that study) for 60 km sections was typically near -3, giving $D \approx 1$. For this reason they concluded that the data were not a fractal set at the scale examined. However, it was observed that at long enough (unspecified) wavelengths, p was generally larger, from which we infer that the fractal characteristics may vary with scale.

Bishop and Chellis [1989] (hereafter BC89) also examined under-ice data collected by submarine sonar, but on the scale of individual relief features such as keels. Their study was based on a data set with points spaced every 0.26 m. Since the draft data may not constitute a stationary random process, the draft increment data, i.e., the difference in draft between pairs of points, may be shown to be stationary and were used instead. Results indicated that the small-scale surface roughness of ice-keels may be parameterized by a fractal dimension ranging from 1.2 to 1.7. *Connors et al.* [1990] estimated fractal dimensions of the ice draft profile of deformed first-year and multiyear ice from power spectra for 256 m segments. Data points were spaced every 0.5 m, and the slope of the spectra was determined in the wavelength range of 1.6 to 16 m. The first-year and multiyear ice segments appeared to be fractal with $D = 1.4$ and 1.6, respectively.

2. Methods

From those studies, we can surmise that the ice draft profile is fractal at small length-scales and may not be fractal over the range of 12-100 m. Does the draft profile exhibit a fractal nature at scales greater than 100 m? In this paper we combine various aspects of the previously described work and extend it in four ways: (1) the under-ice data used here differ in location and time of year and cover a broad range of ice "regions"; (2) the fractal dimension is estimated from the increment data as in BC89 rather than from the power spectra as in RT80; (3) a wide range of scales are examined: 3-15, 15-75, 75-375, and 375-1875 m (factor of 5); and (4) the calculations are done for 20 km sections as opposed to 60 km sections in RT80 and individual keels and non-keel features in BC89. The resolution of the submarine-sonar system precludes the examination of the microscale structure as done in BC89.

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The draft data was collected by the nuclear submarine *USS Queenfish* in August 1970 [*McLaren*, 1989] along 155°W across the Canada Basin, North Pole, and Nansen Basin, for a total track length of over 3000 km. Similar to the data set used by RT80, draft data are sampled at approximately 1.5 m intervals and interpolated (linearly) to 1.5 m spacing, but here the data are not smoothed. However, since the footprint diameter of the sonar beam throughout much of the *Queenfish* track is approximately 3 m (similar to that of the *Gurnard* used in RT80), only one data point every 3 m is used in the analysis. The random error in the measurement system is approximately ±0.1 m. Each 20 km section that had no data gaps is examined. Draft and increment data for 1 km of the track are shown in Figure 1.

To estimate the fractal dimension, the way in which the variance of the increment data changes with lag, Δ, is examined. The variance method has been described for use in image analysis [e.g., *Lundahl et al.*, 1986] and was also used by BC89. This method was favored over spectral analysis because of the difficulty of defining the asymptotic portion of the spectrum and because the original data may not be stationary. The variance of the increments increases with Δ as

$$std[B(t+\Delta) - B(t)] = \Delta^H \, std[B(t+1) - B(t)] \quad (1)$$

where std[] is the standard deviation and $B(t)$ is the draft at point t along the track. The parameter H will be the focus

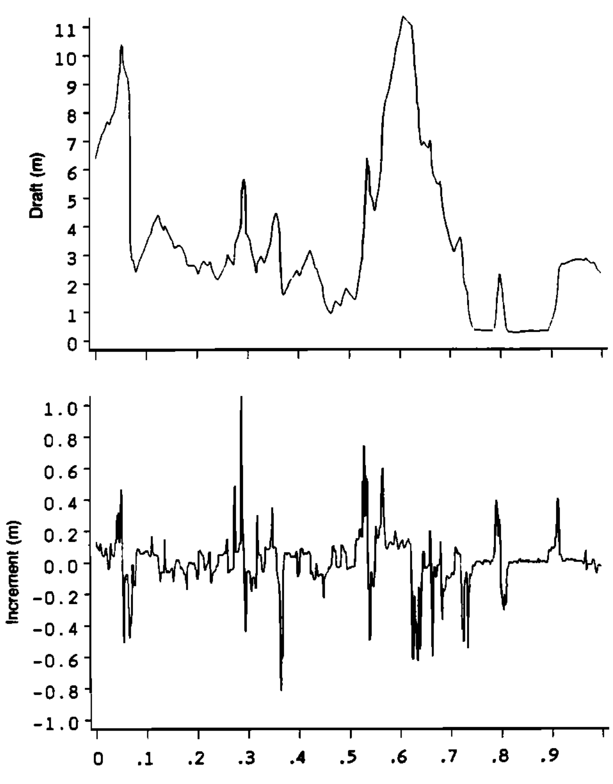

Fig. 1. Draft (top) and draft increment (bottom) data (Δ=3) for 1 km of the submarine track. Increment data are constructed as described in the text.

TABLE 1. Mean and standard deviation of H at various scales.

Scale (m)	Mean	Standard Deviation
3 - 15	0.71	0.06
15 - 75	0.30	0.06
75 - 375	0.06	0.04
375 - 1875	0.03	0.03

of the remaining discussion, and is related to the fractal dimension by $D = 2 - H$ for a self-affine series. "Self-affine" means that a series repeats statistically only when the t and B axes are magnified by different amounts, so that if t is magnified by a factor Δ then B must be magnified by a factor Δ^H, as shown in (1). The estimate of H is obtained from the slope of the least squares regression line when the standard deviation of the increments is plotted as a function of lag in a log-log scale, for some range of lags. A good linear fit (i.e., a correlation greater than 0.95 here) implies that the fractal function (1) correctly approximates the data over the range of scales examined. The special value of H = ½ gives the familiar Brownian motion, or random walk. As a final note on methodology, a process $B(t)$ with spectral density proportional to f^p $(-3 \leq p \leq -1)$, where f is wavenumber, corresponds to $H = -(p+1)/2$.

3. Results

Real surfaces have a finite size that places an upper limit on the applicable scale, while the particle size of which the surface consists sets the lower limit. For sea ice draft measured from submarine sonar, the resolution of the instrument sets the approximate lower limit while the upper limit is determined by the distribution of large-scale features, such as the depths and spacings of keels. Sea ice, on the large scale, is smooth. This large-scale smoothness is inherent in the property of self-affinity since the vertical scale increases by less than the horizontal (H < 1) and the surface flattens. Furthermore, since physical processes usually act over a range of scales, there is no reason to expect that H should be constant on all scales of measurement, and the data will change its fractal characteristics when we pass from one scale of physical process to another. For this reason, we examine H over the range of 3 to 1875 m.

A change in H across scales is typical of the draft profile and is illustrated in Figure 2 and Table 1. The parameter H is shown for two scales: 3-15 and 15-75 m, computed for 20 km sections. The maximum error of the estimate (not shown) is based on the student's t distribution:

$$MEE = t(n-2,\alpha/2) \left[\frac{s_e^2}{\sum_{i=1}^{n} (\Delta_i - \langle\Delta\rangle)^2} \right]^{1/2}$$

where n is the number of increment values, s_e^2 is the variance of the error about the regression line, Δ is lag, $\langle\Delta\rangle$ is the mean lag, and α is the level of significance. In the

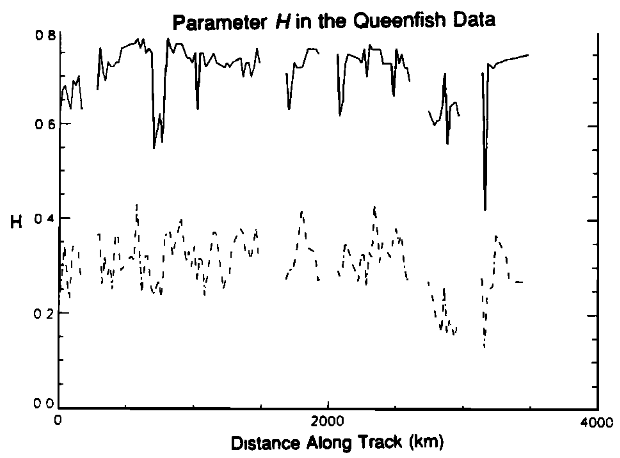

Fig. 2. H at two scales: 3-15 m (solid) and 15-75 m (dashed), computed for 20 km sections. If one or more gaps occurred within the section, no value was computed.

data set, *MEE* ranges from 0.02 to 0.04 at a significance level of 0.05. The confidence interval estimate for the population is $H \pm MEE$; e.g., the 95% confidence interval estimate of H given a sample value of 0.5 is 0.46 to 0.54, based on the largest *MEE* observed. From this analysis, Figure 2 shows that the under-ice draft profile is fractal at both scales but that H varies with scale. However, the validity of the results at the 3-15 m scale over part of the transect is questionable due to the effect of random noise. BC89 suggest that the signal-to-noise ratio, *SNR*, be at least 20 dB for a reliable estimate of H. *SNR* is based on the variances of the increment data, σ_i^2, and the noise (random error) variance, σ_n^2: $SNR = 10 \log(\sigma_i^2/\sigma_n^2)$. For scales greater than 15 m this condition was always satisfied, and for scales shorter than 15 m it was satisfied everywhere except in the central Canada Basin (km 300-2000) where the ice is generally thinner. Consequently, the variance of the increments is small and *SNR* is approximately 10 dB.

The difference in H between the scales suggests that different physical processes are at work, such as ridging and rafting on the large scale and erosion and freezing on the small scale. It has been shown elsewhere [BC89] that at small scales there are both fractal and non-fractal surfaces. The range of H (and therefore D) observed at the smaller scales is within the ranges reported by BC89 and *Connors et al.* [1990], so that adding finer resolution data may not change the results for scales less than about 15 m.

In contrast to our results, the data of RT80 in most cases were not fractal ($p = -3$, so $D = 1$) at scales less than approximately 100 m. This discrepancy may be related to the data set differences or to the methodologies employed. Data set differences are minimal in that the sonar and platform characteristics for *Gurnard* and *Queenfish* were similar. Differences due to season and location are possible, for example a larger open water fraction in the central Canada Basin, but are minimal given the broad range of ice conditions encountered by *Queenfish*. Concerning methodology, the degree of smoothing can have a significant influence on H, with H increasing (D decreasing) as the data are smoothed. This is particularly true for large H, and was observed in the *Queenfish* data, where smoothed increment data exhibited H values 0.2-0.3 smaller than those illustrated for the 3-15 m bin; changes in H for the 15-75 m bin are on the order of 0.1. A similar influence on the fractal dimension estimated from the spectral exponent was observed, where smoothing suppresses the high frequency variability and causes a sharper decrease in the spectral densities at wavelengths less than 100 m, the approximate wavelength range examined in RT80. (The spectral calculations were done with the lag products method using a window of 200 lags, as in RT80.) Indeed, in the *Queenfish* data, D estimated from the power spectra of 20 km sections generally takes on smaller values for both the 3-15 m and 15-75 m scales, with greater variability observed than for D based on the variance of increments method. However, direct application of the frequency property of continuous data to sampled data in spectral analysis is another issue, and *Lundahl et al.* [1986] demonstrate that the discrete power spectrum does not exactly follow this rule, especially for small H ($H < 0.2$). Since H estimated here and in BC89 is in the range of 0.25-0.75, the significance of this is uncertain. Finally, the stationarity within the 60 km sections used in RT80 could be an issue, although an examination of the *Queenfish* data over 50-100 km segments discounts this as a significant factor.

The parameter H in Figure 2 shows no obvious pattern of spatial variability in H over the track. Conversely, Figure 3 illustrates roughness as the standard deviation in ice draft, and clearly shows some regional differentiation, in particular the low variability of the central Canada Basin (km 300-800) and increasing variability towards the North Pole (km 2300) where ice is thicker and more heavily ridged. However, the coefficient of variation shown in Figure 4 exhibits a very limited range along the track, as does H in Figure 2. In this data set, ice regions that are clearly distinguishable by their first-order statistics are not unique in terms of their fractal dimension. Other studies have produced varying results. In RT80, for example, the conclusion was that they were "unable to distinguish nearshore ice from offshore ice on the basis of the shapes of their spectra." *Hibler and LeSchack*

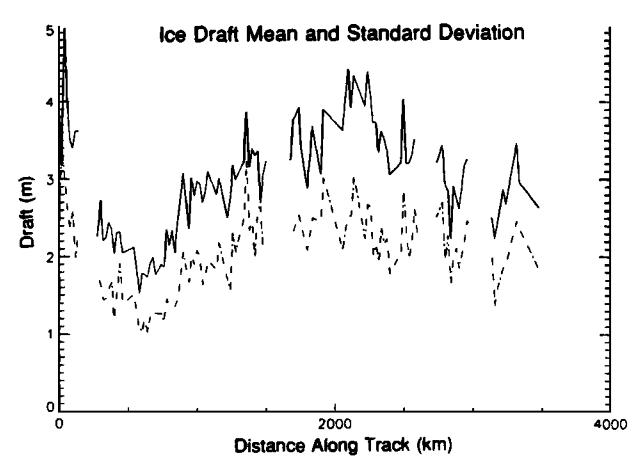

Fig. 3. Mean (solid) and standard deviation (dashed) of ice draft for 20 km sections. Small standard deviations near kilometer 500 correspond to the relatively thin ice of the central Canada Basin.

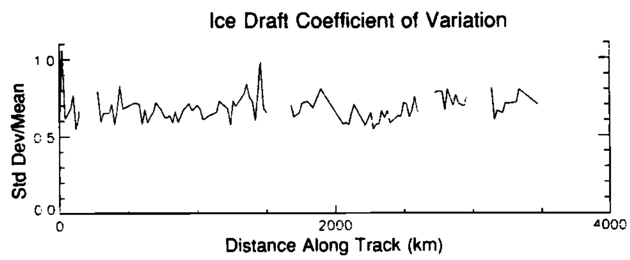

Fig. 4. Coefficient of variation (standard deviation divided by the mean) of ice draft for 20 km sections.

[1972], however, found that the shape of the power spectrum of top-surface laser data seemed to indicate ice type. Similarly, in Figure 9 of *Kozo and Tucker* [1974] there appears to be a change in the slope of the spectra at high frequencies (less than 100 m) as one moves away from the ice edge and toward the Greenland coast. It is not clear if H computed from the data used in these two studies with the variance of increments method would produce similar results.

Finally, we compare the draft data set to a realization of fractional Brownian Motion (FBM), a mathematical generalization of Brownian motion defined by *Mandelbrot and Van Ness* [1968] and further described in *Lundahl et al.* [1986]. In BC89 it was found that some draft profiles may be realizations of FBM. Here samples of the draft increment data from different portions of the transect were tested and found to be realizations of FBM, satisfying the conditions stated in *Lundahl et al.* [1986]. We note, however, that at the scale of individual relief features the determination of stationarity (i.e., identical moments of the distributions of increment data regardless of the origin), a necessary condition of FBM, depends on the length of the segments used. If individual features such as keels are examined, then the data may not be stationary and therefore not a realization of FBM. At the other extreme, the entire track is not stationary because of a trend in the data. To assess the stationarity of the data used here, all lags are examined for 20 km sections in divisions of 5 km, so that the corresponding distributions include many instances of individual features but not trends in the overall track. Defining "identical moments" as means and variances within 20% of each other, the data are stationary.

4. Conclusions

The ice draft data examined constitute a fractal set within a limited range of scales, with $0.55 < H < 0.78$ for scales of 3-15 m and $0.15 < H < 0.45$ for scales ranging from 15-75 m, beyond which H is near unity. However, random error in the data imposes a lower bound on the minimum increments that are meaningful so that the results for the shorter of these two scales are questionable for the Canada Basin portion of the track. From these results it is seen quantitatively that sea ice on the large scale is smooth. The fractal dimension at the smaller scale is similar to that measured by other investigators for individual ice features such as keels. The parameter H did not show any discernible pattern from one region to the next, and is therefore not useful for partitioning the transect into homogeneous ice areas based on second-order roughness statistics in the data set examined. However, draft data sets from other locations need to be examined to see if this holds true in general. Lastly, while the data set consists of draft measurements along a line, these results will be applicable over an area if the underside of the ice is a spatially isotropic surface. While some investigations suggest that the under-ice surface is not spatially isotropic [e.g., *Hibler and LeSchack*, 1972; *Connors et al.*, 1990], side-scan sonar data could be used to confirm this.

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