Cold Regions Science and Technology, 16 (1989) 1–10
Elsevier Science Publishers B.V., Amsterdam — Printed in The Netherlands

PERIODICITIES AND KEEL SPACINGS IN THE UNDER-ICE DRAFT DISTRIBUTION OF THE CANADA BASIN

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(Received May 5, 1988; accepted in revised form August 5, 1988)

ABSTRACT

Previous analyses of the under-ice draft in the Canada Basin have shown this area to be relatively homogeneous in thickness and variability. Spectral analysis was applied to 940 kilometers of under-ice thickness data in the Canada Basin as recorded by the USS Queenfish, August 1970. Periodicities in the range of 30–130 m most commonly occurred throughout the track with those in the ranges of 500–540 m and 800–1000 m also common in areas of thicker ice. Significant periodicities showed little variability throughout the Canada Basin thereby supporting the hypothesis of homogeneity. The spacing distributions of independent keels were compared to the theoretical negative exponential and lognormal distributions and were found to be significantly different from the expected frequencies of the negative exponential model, but generally agreed with the lognormal model except for deep-draft keels, where the distributions did not follow either model.

INTRODUCTION

During early August 1970, the USS Queenfish (SSN-651) continuously recorded the under-ice topography of the Canada Basin along the 155 West meridian between latitudes 74–00.0 and 83–30 degrees North. Subsequent statistical analysis of the acoustically recorded under-ice draft distribution confirmed ship's observations that the under-ice topography between latitudes 76 and 82 degrees North was quite uniform and moderate in draft (McLaren, 1986). McLaren (1986, 1987, 1988) further determined that in summer the sea ice of this area, within the central part of the Beaufort Gyre, may be thinner and more open than elsewhere within the Arctic Basin. The uniformity of ice in the Beaufort Sea was also noted by Wadhams (1980). A study of ice motion in the Canada Basin (McLaren et al., 1987) revealed a recurring tendency for reversal of the mean clockwise gyre in late summer which appears to be associated with divergent ice motion as a result of a low pressure system in the region during late summer.

The above analyses provided the impetus to examine further the under-ice profiles recorded by Queenfish through the Canada Basin (McLaren, 1986) to determine what other indications of environmental forcings might be found. Visual examination of the profile data indicated spatial periodicities. Accordingly, spectral analysis was used to describe the shape and non-randomness of the under-ice draft profiles which Queenfish recorded within this area. This paper presents the results from analyses of periodicities and independent keel spacings in approximately 940 km of under-ice thickness. Our objective was to examine the distributions of periodicities and keel spacings throughout this region, testing the hypothesis of homogeneity.

The morphology of sea ice has been previously analyzed statistically by Rothrock (1979, 1986), Hibler (1980), Thorndike et al. (1975), Wadhams (1981), McLaren et al. (1984), McLaren (1986) and others. Analyses of undersea and surface ice profiles – including spacing of keels and ridges – can be found in Wadhams and Davy (1986), Wadhams

(1983), Wadhams and Horne (1980, 1978), Rothrock and Thorndike (1980), Williams et al. (1975), Hibler et al. (1974), Mock et al. (1972), and Hibler et al. (1972).

Hibler and LeSchack (1972) applied spectral analysis to the undersea ice profile in the central Arctic. They found spacing periodicities of 56 and 82.5 m to be significant. Kozo and Tucker (1974) applied Fourier analysis to sonar data in the Denmark Strait, from the Greenland coast to the ice edge. They found an increase in ice thickness variability with increasing distance from the ice edge and a corresponding increase in the importance of the longer wavelengths (periodicities) in the Fourier analysis.

DATA

Queenfish continuous analog under-ice draft recordings and supporting navigational logs were obtained from the Arctic Submarine Laboratory, U.S. Naval Ocean Systems Center, San Diego. Queenfish used a narrow-beam, 205 kHz, upward-beamed acoustic profiler of an AN/BQS-8 sonar system with a footprint diameter of 2.68 m to record the under-ice topography. The ice draft data on the analog recordings were digitized manually to produce one data point every 0.05 mm of the recording chart. Over three million data points were obtained for subsequent statistical analysis. Since ship's speed variations and manual digitization can produce variable spacing between individual ice draft data points, the data were interpolated to 145 cm intervals, thus assuring a more balanced representation of under-ice thickness. The accuracy of the submarine's acoustic profiler is ±15 cm at best; the precision of the interpolation routine is estimated to be ±1.0 cm.

The study area covers approximately 940 km, between 74°–22.5' N, 155°–00.0' W to 82°–55.0' N, 155°–00.0' W (Fig. 1). Gaps occur in the data, varying in length from 4 to 90 km but account for less than 10% of the study transect. We chose not to fill the gaps; hence these areas were not used in subsequent analyses. The overall track was divided into 5 km subsections with points averaged over 10 m. This combination of segment length and point spacing was chosen because it sufficiently resolves periodicities in the range of interest (20–2000 m) while providing a relatively high degree of homogeneity within each segment and computational efficiency.

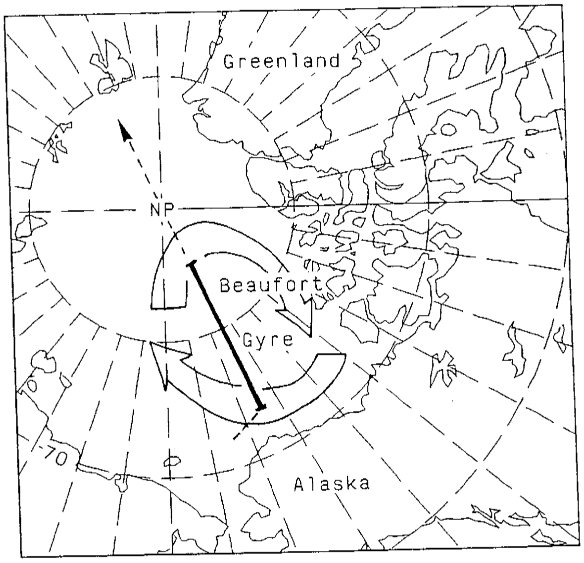

Fig. 1. Track of the USS Queenfish in early August, 1970 across the Canada Basin and the North Pole (dashed and solid lines). The study area encompasses approximately 940 km within the Beaufort Gyre (solid line).

SPECTRAL ANALYSIS OF THE UNDER-ICE PROFILE

Spectral analysis techniques are used to identify cyclical patterns, or periodicities, in data. Spectral analysis theory is well developed; a detailed description may be found in Jenkins and Watts (1958) and Blackman and Tukey (1958). The method applied in this study follows Mitchell et al. (1966), where all serial covariances for lags of zero to half the length of the series are computed and the cosine transforms are then taken, which are the raw estimates of the power spectrum. The *i*th estimate is a measure of the total variance in the original series that is contributed by wavelengths near the *i*th harmonic of the fundamental wavelength. The final spectral estimates are obtained by smoothing the raw estimates with a simple 3-term weighted average.

Since persistence exists in the under-ice series – i.e. the autocorrelation coefficient at lag one of the series, r_1, is significantly different from zero and $r_2 \approx r_1^2; r_1^3 \approx r_1^3$ – the appropriate hypothetical "null" continuum used for significance testing is that of Markov "red noise". The statistic associated with the significance of the deviation from the population is the ratio of the spectral estimate to the population continuum value for each harmonic, which has been found to be distributed as chi-square divided by degrees of freedom (Tukey, 1950). To inhibit aliasing in the analysis, frequencies above the Nyquist frequency were removed with a low-pass, 1–2–1 filter.

Spectral analysis was applied to each 5 km subsection in the track. Figure 2 shows a 5 km under-ice profile from the northern portion of the track. The horizontal axis marks the distance from the beginning of the Queenfish track (the study area begins at km 340). Periodicities of approximately 450 and 300 m are apparent, particularly within the first 2 km. The importance of these periodicities in the explanation of the variance in each profile is illustrated by the corresponding power spectrum shown in Fig. 3. The hypothetical population continuum and the 90% and 95% upper confidence intervals are shown as broken lines. Peaks in the spectrum represent wavelengths of 448, 290, 176, 154, 140, 117, 81, and 71 m. The group of peaks at 140–176 m are better described as a "quasi-periodicity", with the 71 and 81 m wavelength being multiples of these.

Figure 4 represents the same subsection which has been partially reconstructed from six of the statistically significant peaks (95% confidence level) in the spectrum with large amplitudes: 448, 290, 176, 154, 140 and 117 m. These harmonics identify the dominant wavelengths in the data. (We note that, because spectral analysis reveals wave patterns, these periodicities actually represent keel/lead spacings and widths.) This reconstructed series accounts for approximately 30% of the variance in the original series. Adding more harmonics would, as is guaranteed mathematically, result in a more accurate reconstruction of the series. However, this and other such tests demonstrate that only a few harmonics are necessary to realistically represent the dominant wavelenghts in the under-ice draft distribution.

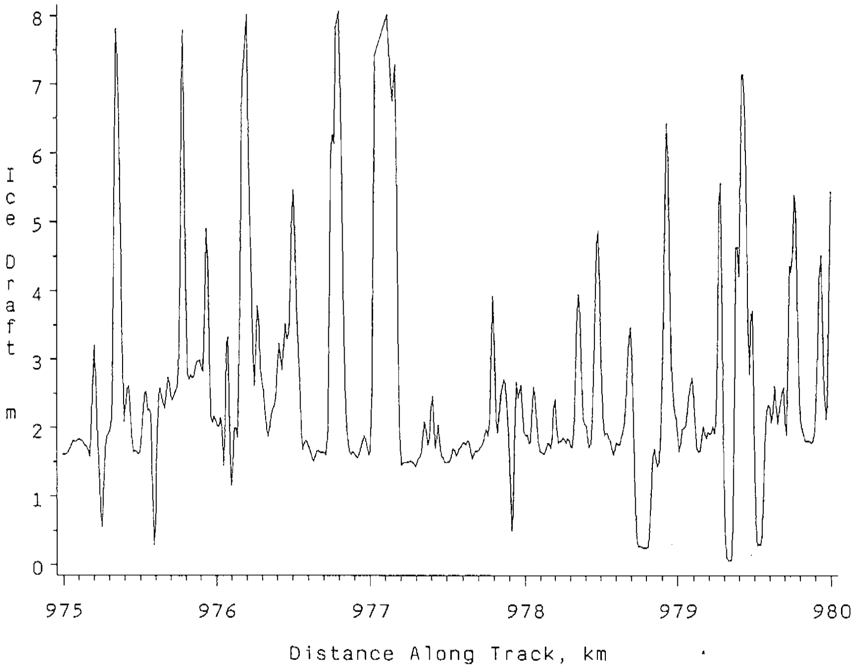

Fig. 2. Under-ice profile of a 5 km section within the study area. The profile is based on the original 1.45 m point spacing averaged over 10 m. The horizontal axis indicates the distance from the beginning of the Queenfish track.

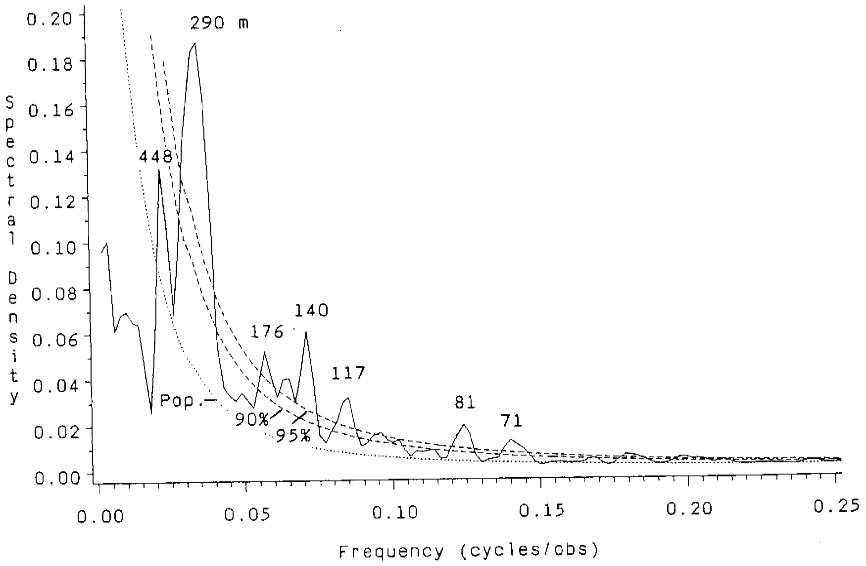

Fig. 3. Power spectrum corresponding to the profile in Fig. 2. Also shown are the population continuum and the 90% and 95% upper confidence levels. Wavelengths of significant peaks are shown in meters.

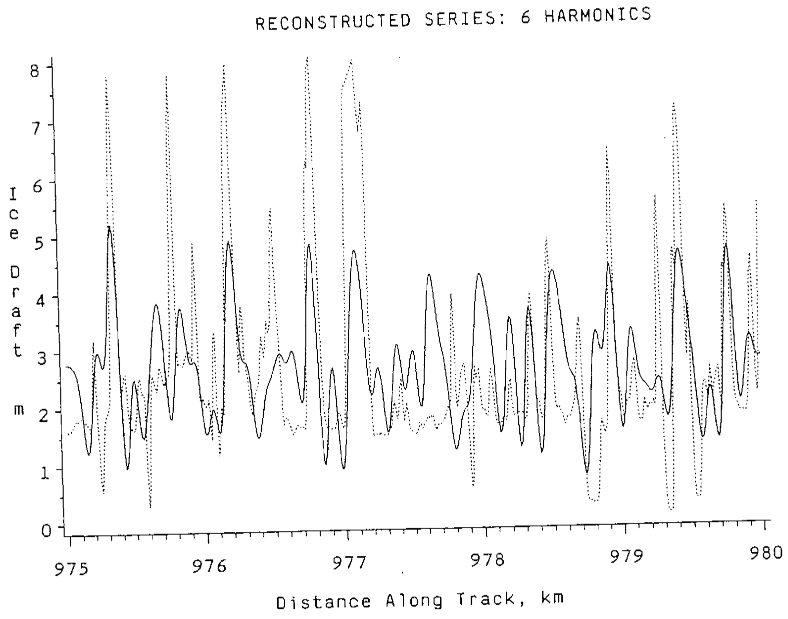

Fig. 4. The profile of Fig. 2 with the original series (broken line) partially reconstructed (solid line) from six harmonics representing 448, 290, 176, 154, 140 and 117 m.

DISTRIBUTIONS OF PERIODICITIES

The procedure described above was applied to each 5 km subsection in the track. While spacings along much of the track vary somewhat randomly from one subsection to the next, some patterns are apparent. Figure 5 summarizes the periodicities for the entire track, where wavelengths corresponding to significant peaks (90% confidence interval) in the spectra were grouped into 20 m bins and frequencies were tabulated. (For the point spacing and subsection length employed in this analysis, harmonics represent spacings of 4920, 2460, 1640, ..., 20 m; therefore gaps occur in the figure). Periodicities in the range of 30–130 m occur most frequently. The shorter of these may correspond to "blisters" observed by Wadhams (1988) in side-scan sonar data which had wavelengths of 28–63 m.

Similar histograms were constructed for five regions along the track (not shown) varying in length from 50 to 140 km (A: 74°–23.0'N to 75°–38.6'N (km 340–480); B: 76°–17.6'N to 77°–39.4'N (km 550–700); C: 78°–18.2'N to 79°–12.2'N (km 775–875); D: 80°–23.0'N to 81°–17.6'N (km 1000–1100); and E: 82°–17.4'N to 82°–44.7'N (km 1210–1260)). These regions correspond roughly to those identified by McLaren (1986). The relative frequency histograms from these regions exhibit the same general shape as that for the overall track. Some regional variations exist; for example, an increase in the frequency wavelengths longer than 1000 m in those areas with greater mean draft and variability, generally beyond kilometer 1000. Similarly, the less variable, thin ice areas show an increase in shorter wavelengths with few greater than 1000 m. Hibler and LeSchack (1972), in a spectral analysis study of the ice surface, also observed that young ice has greater high frequency roughness while multi-year ice is more undulating. Overall, however, the study area exhibits little variability in the periodicities which describe it.

The spatial distribution of subsections along the track which were similar in the periodicities which represent them is shown in Fig. 6. The power spectrum of each subsection was summarized by the amount of variance explained by significant peaks in each of seven wavelength ranges: 50–100; 100–150; 150–250; 250–400; 400–600; 600–800; and 800+ m. Subsections were then clustered (maximum likelihood classifier), where the procedure was constrained to produce five clusters. An examination of the cluster centers for each of seven vari-

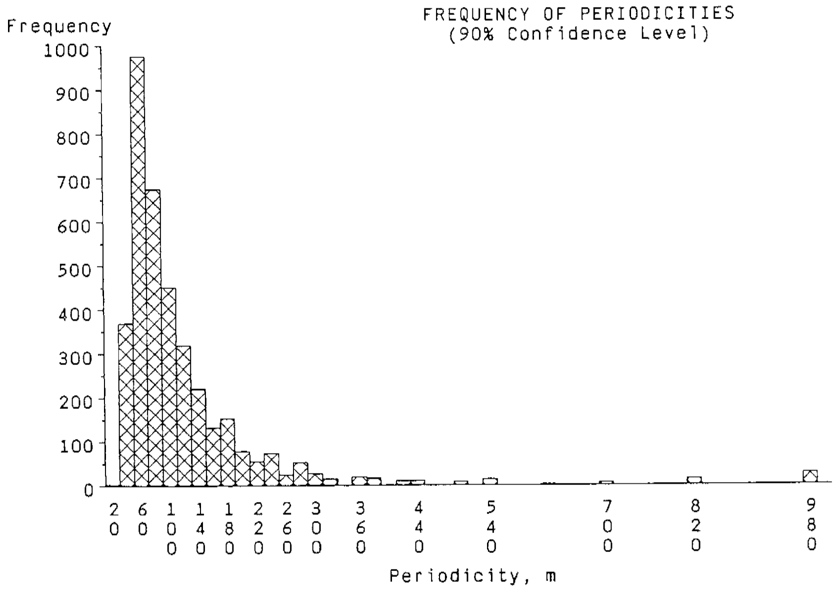

Fig. 5. Summary of the relative frequency of significant (90%) peaks in the power spectra for each five kilometer section in the 940 km study area. Periodicities are grouped in 20 m bins, represented on the horizontal axis by their midpoint.

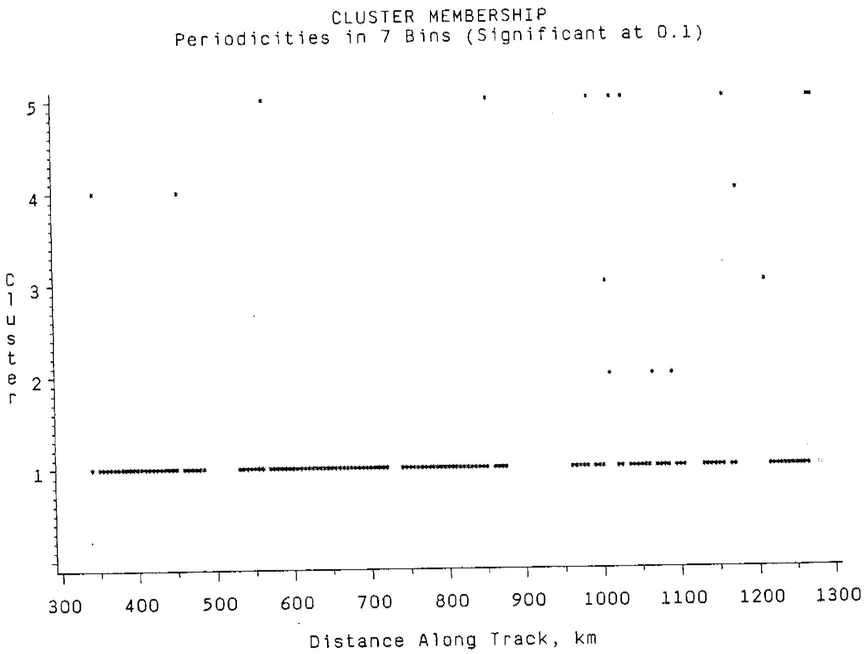

Fig. 6. Cluster membership for each 5 km subsection in the study area. Variables used in cluster analysis are based on variance explained by significant peaks in each of seven wavelength ranges.

ables reveals that the higher cluster numbers represent observations which have a greater amount of power in the longer wavelengths. In general, the first 700 km of the track showed little variability, while the northern 200 km demonstrated an increase in the importance of the longer wavelengths.

The frequency cutoff measure – the number of Fourier components required to account for 95% of the data variance – employed by Kozo and Tucker (1974) was also applied here. In that study, the frequency cutoff ranged from approximately 90 to 180 components, a higher number required in areas of lower standard deviation in ice thickness. The frequency cutoff in the current study averages 102 components throughout the Canada Basin with a range of 90–220, and no trends are apparent in either standard deviation or frequency cutoff. Additionally, only a weak linear correlation exists between the two. We also note that the spectra follow a power law of -1.33 (mean slope of all subsection spectra), exhibiting only a weak relationship with the mean and standard deviation of ice thickness, and the frequency cutoff. These results further support the hypothesis of homogeneity of ice conditions at this place and time.

SPACING DISTRIBUTIONS OF INDEPENDENT KEELS

If the spatial distribution of keel/lead pairs is periodic, spectral analysis may be used to make inferences about keel-to-keel spacings. Spacing distributions of keels may also be examined, without assuming a periodic nature, and the results compared to those of spectral analysis. In this section we examine the concept of independent keels and the relationship between their observed spacing distributions and two theoretical models.

Wadhams and Horne (1980) define an independent keel as one where the troughs on either side of the keel crest (point of local maximum draft) must ascend at least half way towards the local level ice surface, defined arbitrarily as a draft of 2.5 m. This method is also used in Williams et al. (1975) and Wadhams (1981), but differs from Hibler et al. (1974) which specified that the troughs must ascend a fixed distance from the peak. The Wadhams and Horne method is applied here but it is noted that the local level ice surface varies from region to region and should perhaps be a function of the local mean ice thickness and variability rather than an arbitrarily defined constant.

Following Hibler et al. (1972), the distribution of spacings between keels, assuming randomness, is given by

$$P(x)\,dx = \mu \exp(-\mu x)\,dx$$

where μ is the mean number of ridges per unit length of track and $P(x)\,dx$ is the probability that a given spacing lies between x and $(x+dx)$. With this model, the distribution of keel spacings decays exponentially with most keels occurring at shorter spacings. Wadhams and Horne (1980) found this relationship generally holds true for independent keels as well.

Wadhams and Davy (1986), however, reconsidered the negative exponential distribution as a description of spacings and found that a three-parameter lognormal distribution better fit the observed distributions. The lognormal distribution is a modified Gaussian curve wherein the peak is not centered but shifted, emphasizing the smaller spacings. If X is a random variable and there exists a number θ such that the random variable $Z = \ln(X-\theta)$ is normally distributed, then X is said to have a lognormal distribution. The probability density function of X is

$$f(x) = \frac{1}{(x-\theta)\sigma(2\pi)^{\frac{1}{2}}} \exp[-\{\ln(x-\theta)-\mu\}^2/2\sigma^2]$$

where μ and σ are the mean and standard deviation, respectively, of Z (Wadhams and Davy, 1986). Wadhams and Davy found that the threshold, θ, is best chosen as a function of beamwidth. Values of $\theta = 3$ resulted in the best fit for narrow-beam sonar data while $\theta = 10$–15 produced the best results for wide-beam sonar data. This beamwidth dependence is due to the keel shadowing effect and hence the smallest resolvable keel spacing.

These relationships were tested in the study area of the Queenfish track for independent keels with drafts of at least 3, 3.5, 5, and 9 m. As noted by Hibler et al. (1972), the randomness assumption for the theoretical distribution applies to statistically homogeneous areas, so that testing for randomness in non-homogeneous areas may result in mixing distributions. For this reason the five regions defined in the previous section were examined. The original data based on point spacings of 1.45 m were used for these analyses. Table 1 provides the total number of independent keels and the mean number of keels per kilometer for each of these regions and each of the keel draft cutoff values.

A summary of the results is shown in Fig. 7 for three regions where keel spacings are grouped into 20 m bins for keels with drafts of at least 3.5 m. The theoretical negative exponential and lognormal distributions are shown on each histogram. A chi-square goodness-of-fit test was performed under the hypothesis that the observed and theoretical frequency distributions are not significantly different. In all cases the observed frequencies were found to be significantly different (0.05 level) from the negative exponential distribution, implying that spacings are probably not random.

The threshold, θ, in the three-parameter lognormal distribution was varied from 0–18 in increments of 3. The chi-square test was applied for each θ and the linear correlation between the logarithm of each spacing and the logarithm of the cumulative probability up to that spacing was calculated. The correlation steadily increased up to $\theta = 15$ (from approximately 0.85 to 0.90 in most cases) and decreased thereafter. Calculated chi-square values decreased – implying less difference between observed and theoretical distributions – to a minimum of $\theta = 12$, 6, and 9 for regions A, C, and E, respectively.

TABLE 1

Total keels and number of keels per kilometer for each of five regions*

Region		Keel draft cutoff			
		3.0	3.5	5.0	9.0
A	$n=$	942	691	305	53
	$\mu=$	6.7	4.9	2.2	0.4
B	$n=$	626	411	143	15
	$\mu=$	4.2	2.7	1.0	0.1
C	$n=$	660	478	191	18
	$\mu=$	6.6	4.8	1.9	0.2
D	$n=$	972	746	411	69
	$\mu=$	9.7	7.5	4.1	0.7
E	$n=$	463	350	179	33
	$\mu=$	9.26	7.0	3.6	0.7

*See text for exact region boundaries.

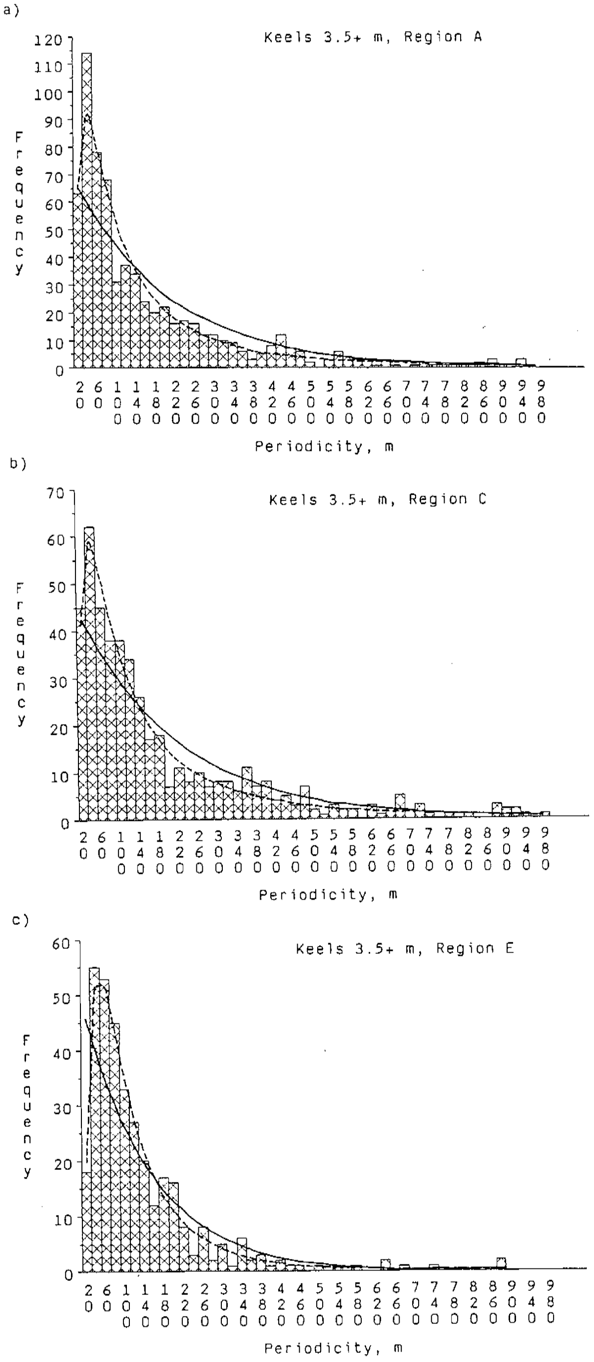

Fig. 7. The frequency distribution of independent keels of at least 3.5 draft in three different regions, approximately located: a) at the beginning (region A); b) middle (region C); and c) end (region E) of the 940 km study area. See text for exact locations. The theoretical negative exponential (solid line) and lognormal curves (dashed line) are also shown. For the lognormal distributions, $\theta = 12$, 6, and 9 for regions, A, C, and E, respectively.

In all cases the data were not found to be significantly different from the lognormal distribution. For all three regions, there are a larger number of observed spacings in the 40–80 m range than expected given a lognormal distribution, even though there is overall agreement between them. This consistent deviation generally agrees with the findings of the spectral analysis where the most frequently occurring periodicities were also in this range.

This analysis was also applied to keels of at least 5 and 9 m draft. The observed distributions for keels of at least 5 m draft were not found to be significantly different from the lognormal distribution with the threshold values as described above. An example for region A is given in Fig. 8. The more uniform distribution illustrated also occurred in the other regions and is assumed to be a function of the relatively mild ice conditions found by Queenfish (McLaren, 1986). This feature is even more exaggerated for keels of at least 9 m, where observed distributions were found to be significantly different from both the negative exponential and lognormal distributions for all regions. Based on this data set, keel spacing distributions cannot be modeled by either probability density function when the threshold value for keel depth is large; e.g. 5 or 9 m. Preliminary investigations of segments with greater mean thickness and variability (e.g. across the North Pole) show a larger number of deepdraft keels per unit length with spacing distributions better modeled by the lognormal distribution. Work by Lowry

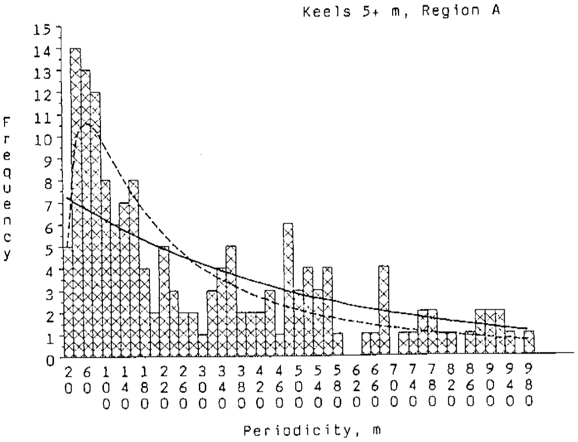

Fig. 8. As with Fig. 7, but for keels of at least five meters draft in region A.

and Wadhams (1979), Wadhams and Davy (1986), and others also describe areas where the mean number of deepdraft keels is higher than that reported here for the Canada Basin, and their corresponding spacing distributions follow the theoretical distribution very closely.

CONCLUDING REMARKS

Spectral analysis has been applied to approximately 940 km of under-ice thickness data as recorded by the narrow-beam sonar of USS Queenfish in the Canada Basin, during early August 1970. To identify spatial periodicities in the profiles, 5 km subsections based on point spacings of 10 m were examined. Periodicities in the range of 30–130 m most commonly occurred throughout the track with those in the ranges of 500–540 m and 800–1000 m also common in the regions of thicker ice. While some regional patterns can be discerned, spectral analysis does not clearly identify regional variations within the study area.

The theoretical negative exponential and lognormal distributions of keel spacings were applied to spacings of independent keels. Frequency distributions observed within five geographical subregions along the track for keels with drafts of at least 3.5 m were found to be significantly different from the expected frequencies as modeled by a negative exponential distribution. Observed data were not found to differ significantly from the three-parameter lognormal model. However, larger than expected numbers of keels spaced at 40–80 m were found in all regions examined.

In this data set, keel spacing distributions cannot be modeled by either probability density function when the threshold value for keel depth is large; e.g. 5 or 9 m. While the lognormal model provides a good fit for observed data when the keel depth threshold is 3 or 3.5 m, this cutoff value is perhaps too small in thick ice areas where thickness can thermodynamically exceed 3 m. We recognize the dependency of the definition of independent keels on their observed spacing distributions and are presently testing definitions which also include keel width and a dynamically determined local level ice surface.

Even though the spectral analysis and the independent keel method do not measure spacings in the same manner, general agreement between them was found. In particular, both methods detected a high frequency of periodicities/spacings in the approximate range of 40–100 m. The distribution of periodicities is also in general agreement with spacing distributions of ridges and independent keels described in Wadhams (1980), Lowry and Wadhams (1979), Hibler et al. (1972), Mock et al. (1972) and others, where the highest frequency of spacings occurs at approximately 100–150 m. Results presented here suggest that models of keel/lead spacings should include a parameter describing their periodic nature. The absence of detailed buoy and satellite data for 1970 and requisite oceanographic and ice mechanics information precludes detailed analysis of possible environmental forcings which produced the observed results. Accordingly, assistance is required from the oceanographic and ice dynamics communities.

Previous analyses of the under-ice draft in the Canada Basin have shown this area to be relatively homogeneous in thickness and variability. Generally periodicities were not found to vary significantly throughout the Canada Basin. Additionally, independent keel spacing distributions were similar in all regions. The hypothesis of homogeneity therefore is not refuted.

Future research included the application of these procedures to other regions within the Queenfish track as well as other submarine tracks. The relationship between geographic location, mean ice thickness and variability, and keel/lead descriptive measures needs to be further investigated. In addition, spectral analyses of sidescan sonar data is planned.

ACKNOWLEDGEMENTS

This work was supported by the Office of Naval Research University Research Initiative Program contract N00014–86–K–0695. We thank R. Bourke, R.G. Barry, J. Walsh, and W.B. Tucker for valuable comments, and Jon Eischeid for programming support.

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